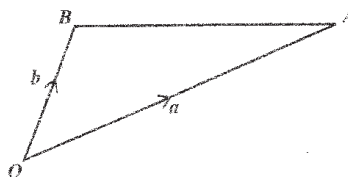## VECTORS AND MATRICES

13.   The position vectors of A and B relative to the origin are a and b respectively.

The point P is on OA such that $OP = 2PA$.
The point M is on BA such that $BM = MA$.

(a)   Copy the diagram and complete it to show the points of P and M.   (2 marks)

(b) OB is produced to N such that $OB = BN$.

   (i)   Show the position of N on your diagram.   (1 mark)

   (ii)   Express in terms of a and b the vectors $\overrightarrow{AB}$, $\overrightarrow{PA}$ and $\overrightarrow{PM}$.   (5 marks)

(c)   Use a vector method to prove that P, M and N are collinear.   (4 marks)

(d)   Calculate the length of AN if

$$a = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ (3 marks)

Total 15 marks

14. (a) X and Y are two matrices where

$$X = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix} \text{ and } Y = \begin{pmatrix} 4 & -1 \\ 3 & 7 \end{pmatrix}.$$

Evaluate $X^2 + Y$. (**4 marks**)

(b) The matrix $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ maps $Q(1, 2)$ to $Q'(5, 7)$.

Find the 2 x 2 matrix which maps Q' back to Q. (**2 marks**)

(c) The vertices of triangle DEF are

$D(5, 12)$, $E(2, 7)$ and $F(8, 4)$.

(i) Triangle DEF undergoes an enlargement with centre, O, and scale factor, k. Its image is $D'E'F'$ where

$D(5, 12) \rightarrow D'(7.5, 18)$.

a) Determine the value of k.

b) Hence write down the coordinates of E' and F'. (**4 marks**)

(ii) $D'E'F'$ undergoes a clockwise rotation of $90°$ about the origin.

a) Determine the 2 x 2 matrix that represents a clockwise rotation of $90°$ about the origin.

b) Determine the coordinates of $D''E''F''$, the image of $D'E'F'$, under this rotation.

c) Determine the 2 x 2 matrix that maps triangle DEF onto triangle $D''E''F''$. (**5 marks**)

Total 15 marks

END OF TEST