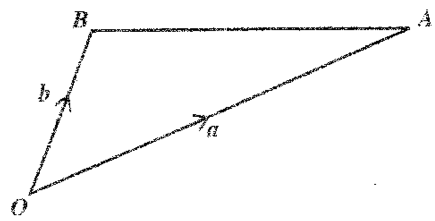
The position vectors of A and B relative to the origin are $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively.



The point P is on OA such that $OP = 2PA$.
The point M is on BA such that $BM = MA$.

(a) Copy the diagram and complete it to show the points of P and M. **(2 marks)**

(b) OB is produced to N such that $OB = BN$.

(i) Show the position of N on your diagram. **(1 mark)**

(ii) Express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ the vectors $\overrightarrow{AB}$, $\overrightarrow{PA}$ and $\overrightarrow{PM}$. **(5 marks)**

(c) Use a vector method to prove that P, M and N are collinear. **(4 marks)**

(d) Calculate the length of AN if

$\boldsymbol{a} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **(3 marks)**

Total 15 marks

(a) X and Y are two matrices where

$$X = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix} \text{ and } Y = \begin{pmatrix} 4 & -1 \\ 3 & 7 \end{pmatrix}.$$

Evaluate $X^2 + Y$. **(4 marks)**

(b) The matrix $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ maps Q (1, 2) to Q'(5, 7).

Find the 2 x 2 matrix which maps Q' back to Q. **(2 marks)**

(c) The vertices of triangle DEF are

D(5, 12), E(2, 7) and F(8, 4).

(i) Triangle DEF undergoes an enlargement with centre, O, and scale factor, k. Its image is $D'E'F'$ where

D (5, 12) $\rightarrow$ D'(7.5, 18).

a) Determine the value of k.

b) Hence write down the coordinates of E' and F'. **(4 marks)**

(ii) $D'E'F'$ undergoes a clockwise rotation of 90° about the origin.

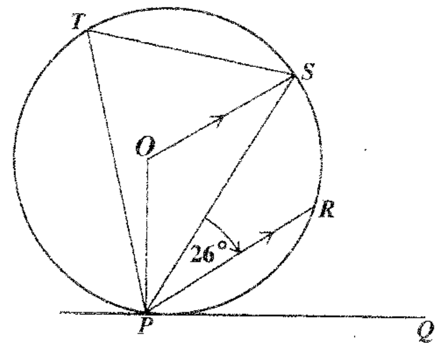
a) Determine the 2 x 2 matrix that represents a clockwise rotation of 90° about the origin.

b) Determine the coordinates of $D''E''F''$, the image of $D'E'F'$, under this rotation.

c) Determine the 2 x 2 matrix that maps triangle DEF onto triangle $D''E''F''$. **(5 marks)**

Total 15 marks

(a) In the diagram below, **not drawn to scale**, PQ is a tangent to the circle, centre O. PR is parallel to OS and angle $SPR = 26°$.

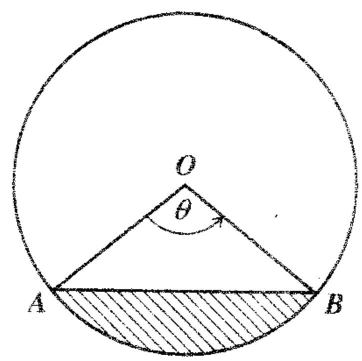


Calculate, **giving reasons for your answer**, the size of

(i) angle PTS **(2 marks)**

(ii) angle RPQ. **(2 marks)**

(b) In the diagram below, **not drawn to scale**, O is the centre of the circle of radius 8.5 cm and AB is a chord of length 14.5 cm.



(i) Calculate the value of θ **to the nearest degree.** **(3 marks)**

(ii) Calculate the area of triangle AOB. **(2 marks)**

(iii) Hence, calculate the area of the shaded region. [Use $\pi = 3.14$]. **(3 marks)**

(iv) Calculate the length of the major arc AB. **(3 marks)**

Total 15 marks

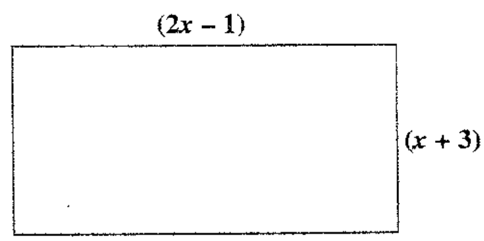
(a) Given that $g(x) = \dfrac{2x + 1}{5}$ and $f(x) = x + 4$.

(i) Calculate the value of $g\ (-2)$.

(ii) Write an expression for $gf(x)$ in its simplest form.

(iii) Find the inverse function $g^{-1}(x)$. **(7 marks)**

(b) The length of the rectangle below is $(2x - 1)$ cm and its width is $(x + 3)$ cm.



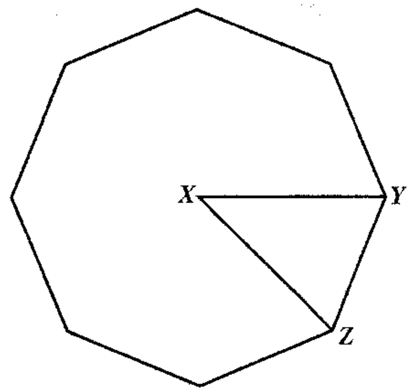
(i) Write an expression in the form $ax^2 + bx + c$ for the area of the rectangle.

(ii) Given that the area of the rectangle is 294 cm^2, determine the value of x.

(iii) Hence, state the dimensions of the rectangle, in centimetres. **(8 marks)**

Total 15 marks

(a) The figure below, **not drawn to scale**, is a regular octagon with centre X, and $XY = 6$ cm.



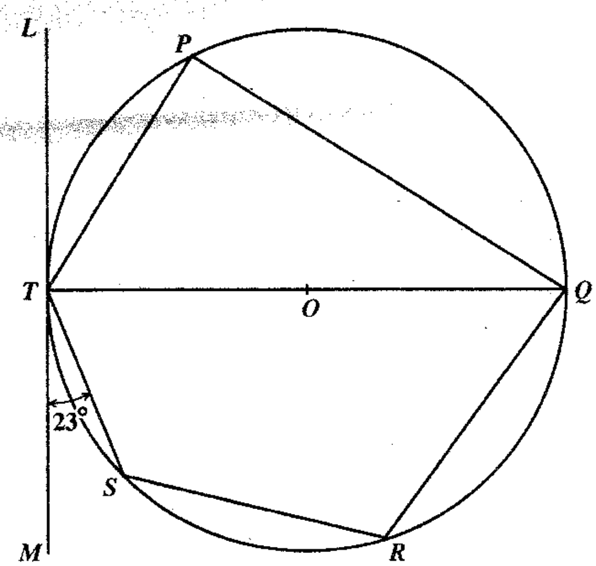
Calculate

(i) the size of angle YXZ

(ii) the area of the triangle YXZ, expressing your answer correct to one decimal place

(iii) the area of the octagon. **(6 marks)**

(b) In the diagram below, **not drawn to scale**, LM is a tangent to the circle at the point, T. O is the centre of the circle and angle $\angle MTS = 23°$.



Calculate the size of each of the following angles, giving reasons for your answer

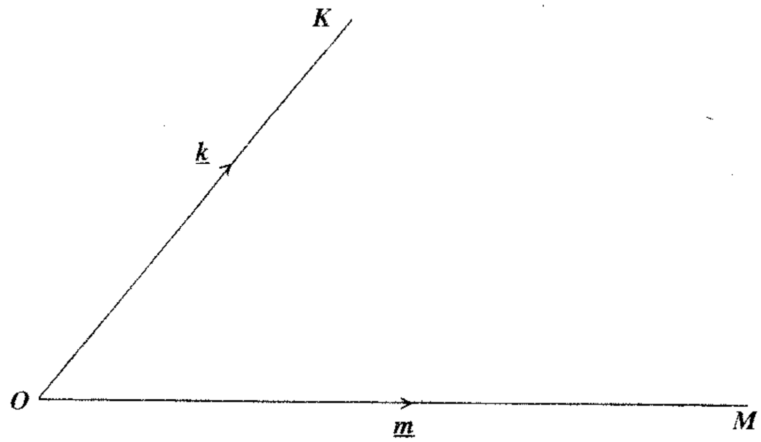
a) angle TPQ

b) angle MTQ

c) angle TQS

d) angle SRQ. **(9 marks)**

Total 15 marks



OK and OM are position vectors such that $\overrightarrow{OK} = \underline{k}$ and $\overrightarrow{OM} = \underline{m}$.

(a) Sketch the diagram above. Show the approximate positions of points R and S such that

R is the mid-point of OK

S is a point on OM such that $\overrightarrow{OS} = \frac{1}{3}\overrightarrow{OM}$. **(2 marks)**

(b) Write down, in terms of $\underline{k}$ and $\underline{m}$ the vectors

(i) $\overrightarrow{MK}$

(ii) $\overrightarrow{RM}$

(iii) $\overrightarrow{KS}$

(iv) $\overrightarrow{RS}$. **(8 marks)**

(c) L is the mid-point of RM. **Using a vector method**, prove that RS is parallel to KL. **(5 marks)**

Total 15 marks

(a) A, B and C are three 2 x 2 matrices such that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $B = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, *and* $C = \begin{pmatrix} 14 & 0 \\ -9 & 5 \end{pmatrix}$.

Find

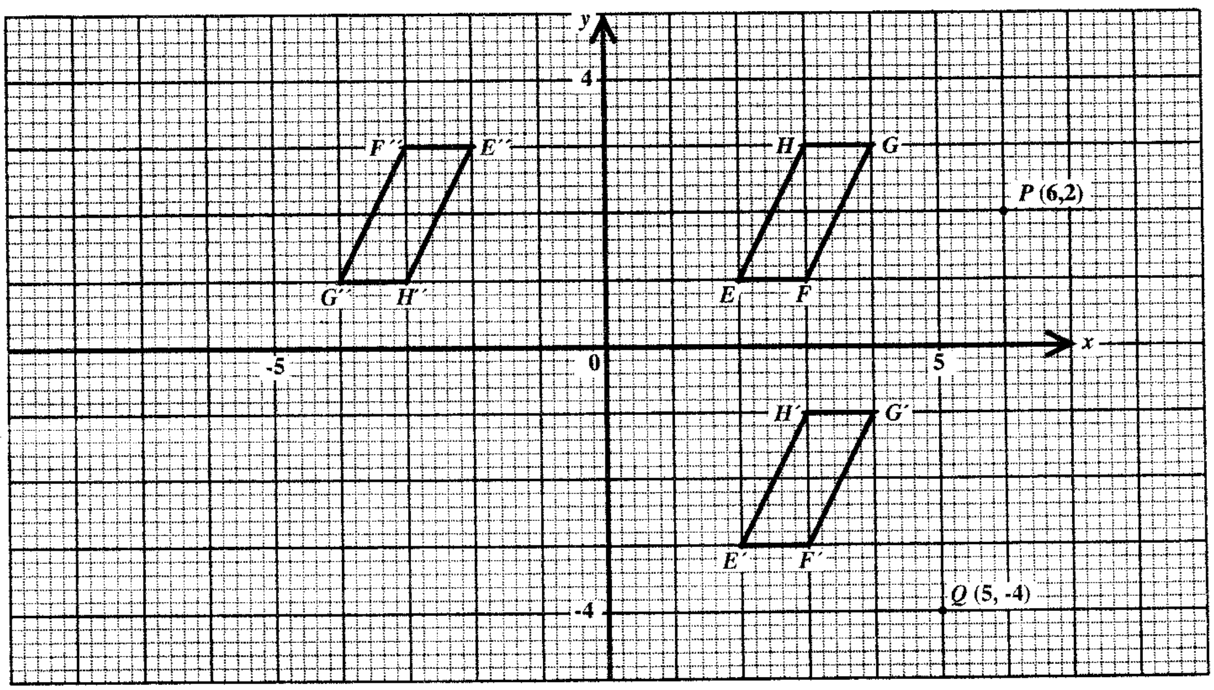
(i) $3A$

(ii) B^{-1}

(iii) $3A + B^{-1}$

(iv) the value of a, b, c and d given that $3A + B^{-1} = C$. **(7 marks)**

(b) The diagram below shows a parallelogram *EFGH* and its images after undergoing two successive transformations.



(i) Describe in words, the geometric transformations

a) J which maps $EFGH$ onto $E'F'G'H'$

b) K which maps $E'F'G'H'$ onto $E''F''G''H''$.

(ii) Write the matrix which represents the transformation described above as

a) J

b) K

(iii) The point P (6, 2) is mapped onto P' by the transformation J. State the co-ordinates of P′.

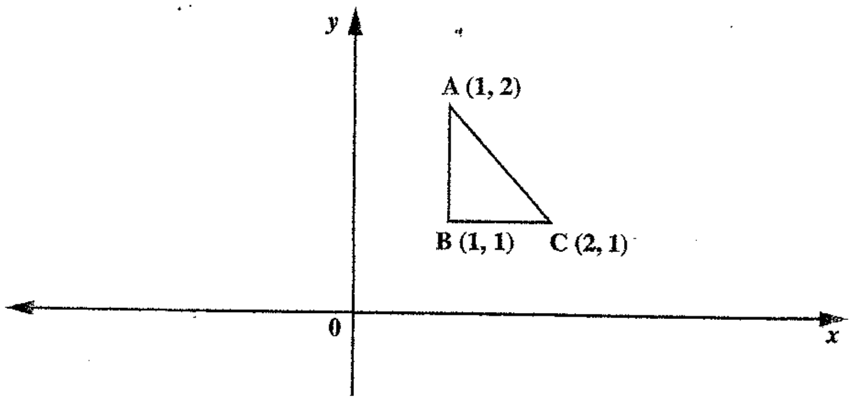
(iv) The point Q (5, −4) is mapped onto Q' by the transformation K. State the co-ordinates of Q′. **(8 marks)**

Total 15 marks

(a) Calculate the matrix product $3AB$, where $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$.

(3 marks)

(b) The diagram below, **not drawn to scale**, shows a triangle, ABC whose coordinates are stated.



Triangle ABC undergoes two successive transformations, V followed by W, where $V = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $W = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) State the effect of V on triangle ABC. **(2 marks)**

(ii) Determine the 2 x 2 matrix that represents the combined transformation of V followed by W. **(3 marks)**

(iii) Using your matrix in (b) (ii), determine the coordinates of the image of triangle ABC under this combined transformation. **(3 marks)**

(c) (i) Write the following simultaneous equations in the form $AX = B$ where A, X and B are matrices:

$$11x + 6y = 6$$
$$9x + 5y = 7$$

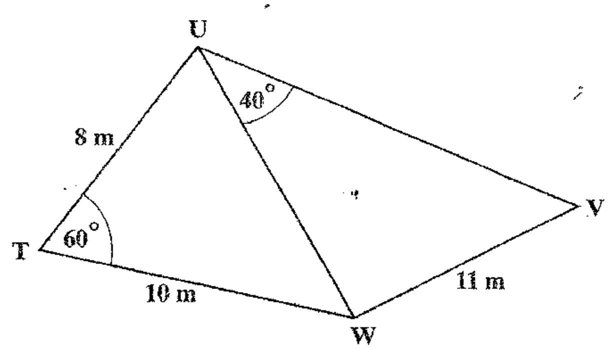
(2 marks)

(ii) **Hence**, write the solution for x and y as a product of two matrices.

(2 marks)

Total 15 marks

(a) On the diagram below, **not drawn to scale**, $TU = 8$ m, $TW = 10$ m, $VW = 11$ m, angle $UTW = 60^\circ$ and angle $WUV = 40^\circ$.



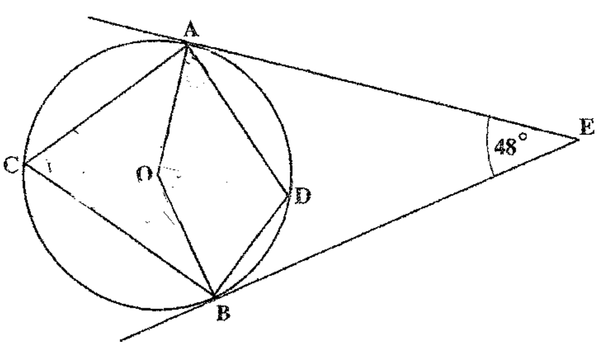
Calculate

(i) the length of UW (2 marks)

(ii) the size of the angle UVW (2 marks)

(iii) the area of triangle TUW. (2 marks)

(b) The diagram below, **not drawn to scale**, shows a circle with centre, O. EA and EB are tangents to the circle, and angle $AEB = 48^\circ$.



Calculate, **giving reasons for your answer**, the size of EACH of the following angles:

(i) $\angle OAE$ **(2 marks)**

(ii) $\angle AOB$ **(3 marks)**

(iii) $\angle ACB$ **(2 marks)**

(iv) $\angle ADB$ **(2 marks)**

Total 15 marks

A ship leaves Port R, sails to Port S and then to Port T.

The bearing of S from R is $112°$.

The bearing of T from S is $033°$.

The distance RT is 75 km and the distance RS is 56 km.

(a) Draw a diagram showing the journey of the ship from R to S to T.
Show on your diagram

(i) the North direction (**1 mark**)

(ii) the bearings $112°$ and $033°$ (**2 marks**)

(iii) the points R, S and T (**1 mark**)

(iv) the distances 75 km and 56 km. (**1 mark**)

(b) Calculate

(i) the size of angle RST (**1 mark**)

(ii) the size of angle RTS (**3 marks**)

(iii) the bearing of R from T. (**2 marks**)

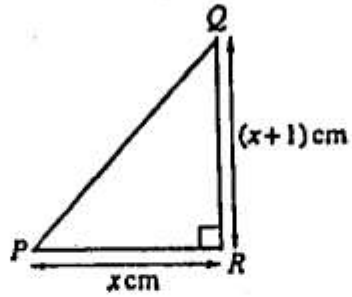
(c) The ship leaves Port T and travels due west to a point X which is due north of R.

(i) Show on your diagram the journey from T to X. (**1 mark**)

(ii) Calculate the distance TX. (**3 marks**)

Total 15 marks

(a)



NOT TO SCALE

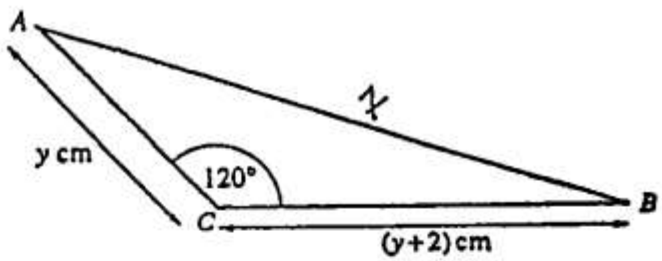
In the right angled triangle PQR, $PR = x$ cm and $QR = (x+1)$ cm.

The area of the triangle PQR is 5 cm^2.

(i) Show that $x^2 + x - 10 = 0$. [3]

(ii) Solve the equation $x^2 + x - 10 = 0$, giving your answers correct to 1 decimal place. Hence write down the length of PR. [6]

(b)



NOT TO SCALE

In triangle ABC, angle $ACB =$ [illegible]°, $AC = y$ cm and $BC = (y+2)$ cm.

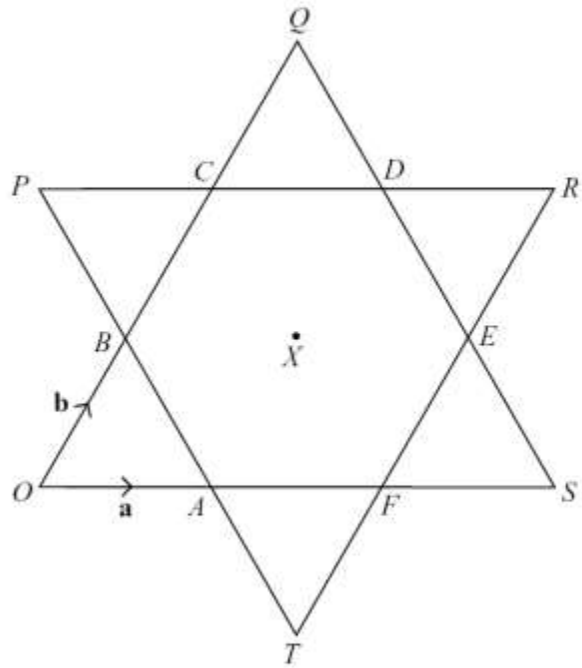
(i) Use the cosine rule to find an expression for AB^2 in terms of y. [2]

(ii) When $AB = 7$ cm, show that $y^2 + 2y - 15 = 0$. [4]

(iii) Factorise $y^2 + 2y - 15$. [1]

(iv) Solve the equation $y^2 + 2y - 15 = 0$.

Hence write down the lengths of AC and CB. [2]



A star is made up of a regular hexagon, centre X, surrounded by 6 equilateral triangles.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Write the following vectors in terms of **a** and/or **b**, giving your answers in their simplest form.

(i) $\overrightarrow{OS}$, [1]

(ii) $\overrightarrow{AB}$, [1]

(iii) $\overrightarrow{CD}$, [1]

(iv) $\overrightarrow{OR}$, [2]

(v) $\overrightarrow{CF}$. [2]

(b) When $|\mathbf{a}| = 5$, write down the value of

(i) $|\mathbf{b}|$, [1]

(ii) $|\mathbf{a} - \mathbf{b}|$. [1]

(c) Describe fully a single transformation which maps

(i) triangle OBA onto triangle OQS, [2]

(ii) triangle OBA onto triangle RDE, with O mapped onto R and B mapped onto D. [2]

(d) **(i)** How many lines of symmetry does the star have? [1]

(ii) When triangle OQS is rotated clockwise about X, it lies on triangle PRT, with O on P.
Write down the angle of rotation. [1]