BINOMIAL EXPANSION THEOREM WORKSHEET

Exercise 1

1. Expand the following binomial expressions:

(a)	$(b+c)^2$	(b)	$(a+g)^3$	(c)	$(1+y)^3$
(d)	$(2+x)^4$	(e)	$(2+2x)^3$	(f)	$(2x-4)^3$
(g)	$\left(2+\frac{y}{7}\right)^4$	(h)	$(2x-5)^3$	(i)	$(3x-4)^3$
(j)	$(3x-9)^3$	(k)	$(2x+6)^3$	(l)	$(b+3d)^3$
(m)	$(3x+2y)^4$	(n)	$(x+3y)^5$	(o)	$\left(2p+\frac{5}{p}\right)^3$
(p)	$\left(x^2-\frac{2}{x}\right)^4$	(q)	$\left(q+\frac{2}{p^3}\right)^5$	(r)	$\left(x+\frac{1}{x}\right)^3$

2. Without expanding the entire expression in number 1 above, find the term indicated for the questions below:

(a) The middle term
(b) The last term written in decreasing powers of *a*
(c) The 3rd term written in increasing powers of *y*
(d) The 3rd term written in decreasing powers of *x*
(e) The 2nd term written in decreasing powers of *x*
(f) The 2nd term written in decreasing powers of *x*
(g) The 4th term written in decreasing powers of *x*
(h) The 3rd term written in increasing powers of *x*
(i) The 4th term written in decreasing powers of *x*
(j) The 1st term written in decreasing powers of *x*
(k) The 3rd term written in increasing powers of *x*
(l) The 2nd term written in decreasing powers of *b*
(m) The 4th term written in decreasing powers of *x*
(n) The 5th term written in increasing powers of *x*
(o) The 2nd term written in decreasing powers of *p*
(p) The 4th term written in decreasing powers of *x*
(q) The 3rd term written in decreasing powers of *p*
(r) The 2nd term written in increasing powers of *x*

Exercise 2

1. Find the terms indicated in the expansions of the following expressions:

	Expression	Term
(a)	$(x+4)^5$	x^3
(b)	$(x+y)^7$	x^5y^2
(c)	$(2x-1)^8$	x^3
(d)	$(3x-2)^5$	x^4
(e)	$(2-3p^2)^4$	p^4
(f)	$(2p-3q)^7$	p^2q^5
(g)	$\left(3p-\frac{2}{p}\right)^7$	p

2. Find the coefficients of the terms indicated in the expansions of the following expressions:

	Expression	Term
(a)	$(2x-5)^8$	x^3
(b)	$(5x-2y)^6$	x^2y^4
(c)	$(x+3)^6$	x^3
(d)	$(2p-3q)^5$	p^4q
(e)	$\left(2x-\frac{3}{p}\right)^8$	$\frac{x^2}{p^6}$
(f)	$\left(q+\frac{2}{p^3}\right)^5$	$\frac{q^3}{p^6}$

3. Use the first three terms in the expansion of $(1+x)^4$ to find an approximate value for 1.01^4. Find the percentage error in using this approximation.

4. (i) Write the expansion of $(5+2x)^6$.

(ii) Use the first three terms of the expansion to approximate 5.2^6.

5. Find the coefficient of x^{-3} in the expansion of $(x-1)^3\left(\frac{1}{x}+x\right)^6$.

6. Find the constant term in the expansion of $\left(x-\frac{1}{2x}\right)^{10}$.

7. Find the constant term in the expansion of $\left(3x-\frac{1}{6x}\right)^{12}$.

8. Find the term independent of x in the expansion of $(2-x)^3\left(\frac{1}{3x}-x\right)^6$.

9. Find the term independent of x in the expansion of $\left(2x-\frac{1}{x}\right)^6\left(\frac{1}{2x}+x\right)^6$.

10. In the expansion of $\left(x-\frac{a}{x}\right)^5\left(x+\frac{a}{x}\right)^5$, where a is a non-zero constant, the coefficient of the term in x^{-2} is '–9' times the coefficient in x^2. Find the value of the constant a.

11. If the coefficient of the x^2 in the expansion of $(1-3x)^n$ is 90, find n.

12. Three consecutive coefficients in the expansion of $(1+x)^n$ are in the ratio 6 : 14 : 21. Find the value of n.

13. Find the independent term in the following expansions

(a) $\left(y+\frac{1}{y}\right)^3\left(y-\frac{1}{y}\right)^5$ (b) $\left(2x+1-\frac{1}{2x^2}\right)^6$

14. In the expansion of $(1+ax)^n$ the first term is 1, the second term is $24x$ and the third term is $252x^2$. Find the values of a and n.

ANSWERS

Exercise 1

1. (a) $b^2+2bc+c^2$ (b) $a^3+3a^2g+3ag^2+g^3$ (c) $1+3y+3y^2+y^3$
(d) $16+32x+24x^2+8x^3+x^4$ (e) $8+24x+24x^2+8x^3$ (f) $8x^3-48x^2+96x-64$
(g) $16+\frac{32}{7}x+\frac{24}{49}x^2+\frac{8}{343}x^3+\frac{1}{2401}x^4$ (h) $8x^3-60x^2+150x-125$
(i) $27x^3-108x^2+144x-64$ (j) $27x^3-243x^2+729x-729$ (k) $8x^3+72x^2+216x+216$
(l) $b^3+9b^2d+27bd^2+27d^3$ (m) $81x^4+216x^3y+216x^2y^2+96xy^3+16y^4$
(n) $x^5+15x^4y+90x^3y^2+270x^2y^3+405xy^4+243y^5$ (o) $\frac{125}{p^3}+\frac{150}{p}+60p+8p^3$
(p) $\frac{16}{x^4}-\frac{32}{x}+24x^2-8x^5+x^8$ (q) $q^8+\frac{10q^4}{p^3}+\frac{40q^3}{p^6}+\frac{80q^2}{p^9}+\frac{80q}{p^{12}}+\frac{32}{p^{15}}$ (r) $x^5+3x+\frac{3}{x}+\frac{1}{x^3}$

2. see above expansion for indicated term

Exercise 2

1. (a) $160x^3$ (b) $21x^5y^2$ (c) $-448x^3$ (d) $-810x^4$ (e) $216p^3$ (f) $-20412p^2q^5$ (g) $-22680p$
2. (a) -1400000 (b) 6000 (c) 540 (d) -240 (e) 81648 (f) 40 **3.** 1.0406 0.0004%
4. i. $64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625$ ii. 19750 iii. 20.6
iv. 0.1% **5.** 19 **6.** $-\frac{63}{8}$ **7.** $\frac{231}{16}$ **8.** $-\frac{130}{27}$ **9.** –20 **10.** $a=\pm3$ **11.** $n=5$ **12.** $n=9$
13. (a) 0 (b) –59 **14.** $a=3, n=8$ **15.** $a=\pm2, b=\pm1$