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Web Resources

Video on how to solve equations with rational exponents : www.mathwarehouse.com/algebra/exponents/equations/solve-exponential-equations-radical-in-exponent.php

Graph Paper Maker (free): www.mathworksheetsgo.com/paper/

Online Graphing Calculator(free): www.mathworksheetsgo.com/calculator/

Solving Equations with Rational Exponents

For any real number *x* and *m* and *n* are natural numbers:

$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ and $a^{-\frac{m}{n}} = \left(\frac{1}{\sqrt[n]{x}}\right)^m$, *a* cannot be negative if *n* is even

Recall: $(x^a)^b = x^{ab}$ for any real number *x*, *a*, and *b*.

I. Model Problems

In these examples we will practice solving equations with rational exponents.

Example 1: **Solve $x^{\frac{2}{3}} - 2 = 7$**

Isolate the variable term.

$$x^{\frac{2}{3}} - 2 + 2 = 7 + 2$$
$$x^{\frac{2}{3}} = 9$$

Raise to the reciprocal power to get x^1.

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (9)^{\frac{3}{2}}$$

Rewrite.

$$x = \left(\sqrt{9}\right)^3$$

Simplify.

$$x = 27$$

Substitute and check for extraneous solutions.

$$(27)^{\frac{2}{3}} - 2 = 7$$
$$\left(\sqrt[3]{27}\right)^2 - 2 = 7$$
$$(3)^2 - 2 = 7$$
$$9 - 2 = 7$$

Answer: $x = 27$

Example 2: **Solve $-2x^{\frac{3}{2}} = 16$**

Isolate the variable term.

$$\frac{-2x^{\frac{3}{2}}}{-2} = \frac{16}{-2}$$
$$x^{\frac{3}{2}} = -8$$

Raise to the reciprocal power to get x^1.

$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (-8)^{\frac{2}{3}}$$

Rewrite.

$$x = \left(\sqrt[3]{-8}\right)^2$$

Simplify.

$$x = (-2)^2$$
$$x = 4$$

Substitute and check for extraneous solutions.

$$-2(4)^{\frac{3}{2}} = 16$$
$$-2\left(\sqrt{4}\right)^3 = 16$$
$$-2\,(2)^3 = 16$$
$$-2(8) = 16$$
$$-16 \neq 16$$

$x = 4$ is an extraneous solution.

Answer: no solution

II. Practice Problems

Solve.

1. $x^{\frac{1}{2}} = 7$
2. $x^{\frac{2}{3}} = 64$
3. $x^{-\frac{1}{4}} = 5$
4. $x^{-\frac{3}{5}} = -27$
5. $x^{\frac{4}{7}} = 16$
6. $x^{\frac{1}{2}} = 7$
7. $x^{\frac{3}{2}} - 6 = -5$
8. $x^{\frac{2}{5}} + 13 = 9$
9. $x^{-\frac{5}{2}} - 57 = 186$
10. $x^{\frac{3}{2}} + 12 = -52$
11. $5x^{\frac{4}{3}} = 80$
12. $\frac{3}{5}x^{-\frac{2}{3}} = \frac{3}{20}$
13. $-15x^{\frac{3}{7}} = 405$
14. $\frac{x^{\frac{3}{4}}}{2} = -64$
15. $8x^{\frac{2}{3}} = 16$
16. $3x^{\frac{3}{2}} + 25 = 400$
17. $-4x^{\frac{5}{3}} + 75 = 203$
18. $4x^{\frac{2}{3}} + 7 = 55$
19. $\frac{1}{5}x^{\frac{3}{4}} - 31 = -6$
20. $-6x^{-\frac{3}{2}} + 5 = -2053$

III. Challenge Problems

1. The braking distance, *d*, in feet of a car traveling with a velocity, *v*, in miles per hour can be found with the equation $d = 0.02v^{\frac{7}{3}}$. If it takes the car 327.68 feet to come to a complete shop, how fast was the car traveling?

2. The number of automobiles a manufacturer can produce, *N*, using *x* units of labor and *y* units of capital can be found with the equation: $N = 60x^{\frac{1}{2}}y^{\frac{1}{2}}$. If 36,000 automobiles are produced with 400 units of labor, how many units of capital are needed?

3. Find the error in the student's work:

$$x^{\frac{3}{2}} = 27$$
$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (27)^{\frac{2}{3}}$$
$$x = 18$$

4. Find the error in the student's work:

$$x^{-\frac{1}{2}} = 4$$
$$\left(x^{-\frac{1}{2}}\right)^{-2} = (4)^{-2}$$
$$x = -16$$

5. Rewrite the following equation with rational exponents:

$$-\left(\sqrt[5]{x}\right)^4 + 16 = 27 + \left(\sqrt[5]{-x}\right)^3$$

IV. Answer Key

1. $x = 49$
2. $x = 64$
3. $x = \frac{1}{625}$
4. $x = -\frac{1}{242}$
5. $x = 128$
6. $x = 243$
7. $x = 1$
8. no solution
9. $x = \frac{1}{9}$
10. no solution
11. $x = 8$
12. $x = 8$
13. $x = -2187$
14. no solution
15. $x = 2\sqrt{2}$
16. $x = 25$
17. $x = -8$
18. $x = 24\sqrt{3}$
19. $x = 625$
20. $x = \frac{1}{49}$

Challenge Problems

1. 64 mph
2. 900 units
3. multiplied 27 by $\frac{2}{3}$ rather than raising 27 to the $\frac{2}{3}$ power; answer should be $x = 9$
4. Raising to a negative power means find the reciprocal; the negative in the exponent does not affect the sign of the number. The answer should be $x = \frac{1}{16}$
5. $-\left(x^{\frac{4}{5}}\right) + 16 = 27 + (-x)^{\frac{5}{5}}$