ALGEBRA

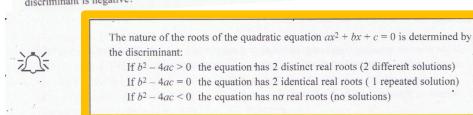
QUADRATIC EQUATIONS -DISCRIMINANT PROPERTIES

Notice the expression under the square root sign of the quadratic formula: $b^2 - 4ac$. This is known as the discriminant, sometimes abbreviated to Δ .

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For the quadratic function ax^2 + bx + c
the expression \Delta = b^2 - 4ac is known as the discriminant.
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The solutions of an equation are known as its roots. What happens to the roots of a quadratic equation when the discriminant is negative?



Example

Determine whether or not the following equations have real roots, and if so, find them: . (a) $x^2 + 6x + 11 = 0$ (b) $2x^2 - 3x - 3 = 0$ (c) $x^2 - 10x + 25 = 0$

- (a) $\Delta = b^2 4ac^2 = 36 44 < 0$... equation does not have real roots
- (b) $\Delta = b^2 4ac^2 = 9 + 24 > 0$
 - ... equation has real roots which are

 $x = \frac{3 \pm \sqrt{33}}{4}$ i.e. 2.19 and -0.686

(c) $\Delta = b^2 - 4ac^2 = 100 - 100 = 0$

 \therefore equation has identical roots given by $(x - 5)^2 = 0$, i.e. x = 5

EXERCISE

1.	Without solving these equations, find whether each has real unequal roots, real equal roots, or no real roots:	
	(a) $x^2 + 4x + 1 = 0$ (b) $x^2 - 3x + 3 = 0$ (c) $x^2 - 8x + 16 = 0$	
	(d) $2x^2 + 3x - 5 = 0$ (e) $4x^2 + 12x + 9 = 0$ (f) $3x^2 - 5x + 3 = 0$	
2.	Find the two possible values of p if the equation $4x^2 - (p-2)x + (p+3) = 0$ has equal roots, and find the root in each case.	
3.	Find the range of possible values of k if the equation $x^2 - 6x + k = 0$ has real roots.	
4.	Find the range of values of p if the equation $(x-3)(x-p) = p(1-x)$ has real roots. Find also the value of p for which the equation has equal roots, and find the root in this case.	
5.	Find the range of values of h for which the equation $(h-3)x^2 + 2hx + (h-1) = 0$ has no real roots.	
#6.	Show that the equation $x^2 + (q + 4)x + (3 + q) = 0$ has real roots for all values of q. Show also that one root is independent of the value of q, and find this root.	

ANSWERS

1.	(a) real, unequal
	(b) none
	(c) real, equal
	(d) real, unequal
	(e) real, equal
	(f) none
2.	$p = -2, x = -\frac{1}{2}; p = 22, x = 2\frac{1}{2}$
3.	$k \leq 9$
4.	$p \le 1\frac{1}{8}; \ p = 1\frac{1}{8}, \ x = 1\frac{1}{2}$
5.	$h < \frac{3}{4}$
6.	(proof) $x = 2$