ALGEBRA

QUADRATIC EQUATIONS -DISCRIMINANT PROPERTIES

Notice the expression under the square root sign of the quadratic formula: $b^2 - 4ac$. This is known as the discriminant, sometimes abbreviated to Δ.

For the quadratic function $ax^2 + bx + c$
the expression $\Delta = b^2 - 4ac$ is known as the **discriminant**.

The solutions of an equation are known as its roots. What happens to the roots of a quadratic equation when the discriminant is negative?

The nature of the roots of the quadratic equation $ax^2 + bx + c = 0$ is determined by the discriminant:

If $b^2 - 4ac > 0$ the equation has 2 distinct real roots (2 different solutions)
If $b^2 - 4ac = 0$ the equation has 2 identical real roots (1 repeated solution)
If $b^2 - 4ac < 0$ the equation has no real roots (no solutions)

Example

Determine whether or not the following equations have real roots, and if so, find them:
(a) $x^2 + 6x + 11 = 0$ (b) $2x^2 - 3x - 3 = 0$ (c) $x^2 - 10x + 25 = 0$

(a) Δ = "$b^2 - 4ac$" $= 36 - 44 < 0$
$\therefore$ equation does not have real roots

(b) Δ = "$b^2 - 4ac$" $= 9 + 24 > 0$
$\therefore$ equation has real roots which are

$$x = \frac{3 \pm \sqrt{33}}{4}$$ i.e. 2.19 and –0.686

(c) Δ = "$b^2 - 4ac$" $= 100 - 100 = 0$
$\therefore$ equation has identical roots given by $(x - 5)^2 = 0$, i.e. $x = 5$

EXERCISE

1. Without solving these equations, find whether each has real unequal roots, real equal roots, or no real roots:
   (a) $x^2 + 4x + 1 = 0$ (b) $x^2 - 3x + 3 = 0$ (c) $x^2 - 8x + 16 = 0$
   (d) $2x^2 + 3x - 5 = 0$ (e) $4x^2 + 12x + 9 = 0$ (f) $3x^2 - 5x + 3 = 0$

2. Find the two possible values of p if the equation $4x^2 - (p - 2)x + (p + 3) = 0$ has equal roots, and find the root in each case.

3. Find the range of possible values of k if the equation $x^2 - 6x + k = 0$ has real roots.

4. Find the range of values of p if the equation $(x - 3)(x - p) = p(1 - x)$ has real roots.
   Find also the value of p for which the equation has equal roots, and find the root in this case.

5. Find the range of values of h for which the equation $(h - 3)x^2 + 2hx + (h - 1) = 0$ has no real roots.

6. Show that the equation $x^2 + (q + 4)x + (3 + q) = 0$ has real roots for all values of q. Show also that one root is independent of the value of q, and find this root.

ANSWERS

1. **(a)** real, unequal
   (b) none
   (c) real, equal
   (d) real, unequal
   (e) real, equal
   (f) none

2. $p = -2,\ x = -\frac{1}{2};\ p = 22,\ x = 2\frac{1}{2}$

3. $k \leq 9$

4. $p \leq 1\frac{1}{8};\ p = 1\frac{1}{8},\ x = 1\frac{1}{2}$

5. $h < \frac{3}{4}$

6. (proof) $x = 2$