ALGEBRA CONCEPT CHECK
(with answers)

Expansion- *Expand and simplify:*

1. $(2x - 5)(3x + 4)$
2. $(2x - 5)(2x + 5)$
3. $(2x - 5)^2$

Factorization- *Factorize fully:*

4. $2x - 10$
5. $12x^2 + 3x - 6x^3$
6. $6x^3 + 2x^2 - 15x - 5$
7. $12x^3 - 75x$
8. $6x^2 - 11x + 3$

Completing the Square- *Write the following in the form* $\underline{a(x + p)^2 + q}$

9. $x^2 - 9x + 5$
10. $-6x^2 - 7x + 3$

Simplifying Algebraic Fraction- *Simplify the following*

11. $\frac{5x-1}{x^2-9} - \frac{2x-7}{x+3}$
12. $\frac{5x-15x^2}{6x^2+16x-6}$

Indices – *Evaluate the following:*

13. $4^0 - 4^{-1} + 4^{0.5}$
14. $25^{3/2}$
15. $\left(\frac{27}{125}\right)^{-2/3}$

Transposition- *make x the subject of the formulae below:*

16. $5y - 3x^2 = 3w$
17. $\sqrt{\frac{4t}{5-6x}} = 3v$
18. $\frac{5x-y}{x-9} = 3z$
19. $\frac{\sqrt{5+3x^2}}{2x} = 3y$

Linear Equations- *solve the following:*

20. $6 - 4(3x - 2) = 6x - (2x + 1)$
21. $\frac{2x+3}{4} - \frac{4x-5}{6} = 5$

Quadratic Equations- *solve the following:*

22. $-3x^4 + 18x^3 = 15x^2$
23. $\frac{4}{3y+4} = -1 + \frac{4}{y}$
24. $7t^2 - 15t + 6 = 4$
25. $-8x^2 - 7x + 3 = 0$ (give ans to 3 d.p)
26. $x^2 - 9x + 5 = 0$ (by completing the square)
27. $-6x^2 - 7x + 3 = 0$ (by completing the square)

Simultaneous Equations- *Solve the following:*

28. $\frac{x-2y}{8} = \frac{1}{2}$

$3x + 2y = 4$

29. $2x^2 - 3y^2 = 5xy$

$-3x + y = 5$

Indical/Power Equations – *solve*

30. $\frac{27^{2x-3}}{9^{(x+4)}} = 3$

Variation

31. x varies with the square y. When $x = 62.5$, $y = 5$
    a) Find y when $x = \frac{125}{72}$
    b) State what happens to x when y is doubled
    c) What percentage change occurs in x when y is increased by 30%

32. x varies inversely with the cube y. When $x = 0.75$, $y = 2$
    a) Find x when $y = -3$
    b) State what happens to x when y is doubled
    c) What percentage change occurs in x when y is decreased by 10%
    d) decreased by 10%

1. $6x^2 - 7x - 20$

2. $4x^2 - 25$

3. $4x^2 - 20x + 25$

4. $2(x - 5)$

5. $3x(4x + 1 - 2x^2)$

6. $(2x^2 - 5)(3x + 1)$

7. $3x(2x - 5)(2x + 5)$

8. $(3x - 1)(2x - 3)$

9. $\left(x - \frac{9}{2}\right)^2 - 15\frac{1}{4}$

10. $-6\left(x + \frac{7}{12}\right)^2 + \frac{121}{24}$

11. $\dfrac{-2x^2 + 18x - 22}{x^2 - 9}$

12. $\dfrac{5x\left(1 - \frac{1}{3}x\right)}{2(3x - 1)(x + 3)} = \dfrac{-5x}{2(x + 3)}$

13. $1 - \frac{1}{4} + 2 = 2.75$

14. 125

15. $\dfrac{25}{9}$

16.

16. $5y - 3x^2 = 3w$

$-3x^2 = 3w - 5y$

$x^2 = \dfrac{3w - 5y}{-3}$

$x = \pm\sqrt{\dfrac{3w - 5y}{-3}}$

17. $\sqrt{\dfrac{4t}{5 - 6x}} = 3v$

$\dfrac{4t}{5 - 6x} = 9v^2$

$4t = 45v^2 - 54xv^2$

$\dfrac{54xv^2}{54v^2} = \dfrac{45v^2 - 4t}{54v^2}$

$x = \dfrac{45v^2 - 4t}{54v^2}$

18. $\dfrac{5x - y}{x - 9} = 3z$

$5x - y = 3xz - 27z$

$5x - 3xz = -y - 27z$

$x(5 - 3z) = -y - 27z$

$x = \dfrac{-y - 27z}{5 - 3z}$

or

$x = \dfrac{y + 27z}{3z - 5}$

19.

$$\frac{\sqrt{5+3x^2}}{2x} = 3y$$

$$\sqrt{5+3x^2} = 6xy$$

$$5 + 3x^2 = 36x^2y^2$$

$$3x^2 - 36x^2y^2 = -5$$

$$x^2(3 - 36y^2) = -5$$

$$x^2 = \frac{-5}{3-36y^2}$$

$$x = \pm\sqrt{\frac{-5}{3-36y^2}}$$

or

$$x = \pm\sqrt{\frac{5}{36y^2-3}}$$

Linear Equations

20. $6 - 4(3x-2) = 6x - (2x+1)$

$$6 - 12x + 8 = 6x - 2x -$$
$$14 - 12x = 4x - 1$$
$$14 + 1 = 12x + 4x$$
$$15 = 16x$$
$$x = \frac{15}{16}$$

21. $\frac{2x+3}{4} - \frac{4x-5}{6} = 5$

$$\frac{3(2x+3) - 2(4x-5)}{12} = 5$$
$$6x + 9 - 8x + 10 = 60$$
$$-2x = 41$$
$$x = -\frac{41}{2}$$

22. $-3x^4 + 18x^3 = 15x^2$

$$-3x^2(x^2 - 6x - 5) = 0$$
$$-3x^2(x-5)(x-1) = 0$$

Either Or Or

$-3x^2 = 0$ $x-5=0$ $x-1=0$

$x = 0$ $x = 5$ $x = 1$

23. $\frac{4}{3y+4} = -1 + \frac{4}{y}$

$$\frac{4}{3y+4} = \frac{-y+4}{y}$$
$$4y = (3y+4)(-y+4)$$
$$4y = -3y^2 + 12y - 4y + 16$$

$$3y^2 - 4y - 16 = 0$$

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\times3\times-16}}{2\times3}$$

$$y = \frac{4 \pm \sqrt{16+192}}{6}$$

$$y = \frac{4 \pm \sqrt{198}}{6}$$

Either Or

$y = 3.01$ $y = -1.68$

24. $7t^2 - 15t + 6 = 4$

   $7t^2 - 15t + 2 = 0$

   $(7t - 1)(t - 2) = 0$

   Either   or

   $t = 1/7$ $t = 2$

25. $-8x^2 - 7x + 3 = 0$ (to 3 dp)

Give ans to 3 d.p suggest that factorization will not work so you must use the formula

$x = \frac{-(-7)\pm\sqrt{(-7)^2-(4\times-8\times3)}}{2\times-8}$ $x = 0.3315 \; , -1.190$

26. $x^2 - 9x + 5 = 0$ (by completing the square)

   $c = \left(\frac{b}{2a}\right)^2$

   $x^2 - 9x + \left(\frac{-9}{2}\right)^2 = -5 + \left(\frac{-9}{2}\right)^2$

   $(x - \frac{9}{2})^2 = 15\,^1/_4$

   $x = \frac{9}{2} \pm \sqrt{15\,^1/_4}$

27. $-6x^2 - 7x + 3 = 0$

   $c = \left(\frac{b}{2a}\right)^2$

   $\{\div -6\}$ $x^2 + \frac{7}{6}x - \frac{3}{6} = 0$

   $x^2 + \frac{7}{6}x + \left(\frac{7}{12}\right)^2 = \frac{1}{2} + \left(\frac{7}{12}\right)^2$

   $(x - \frac{7}{12})^2 = \,^{121}/_{144}$

   $x = \frac{7}{12} \pm \sqrt{\,^{121}/_{144}}$

   $x = \frac{1}{3}$ or $x = -\frac{3}{2}$

28

$$\frac{x-2y}{8} = \frac{1}{2}$$

$$2x - 4y = 8 \quad ①$$

$$3x + 2y = 4 \quad ② \times 2$$

$$2x - 4y = 8$$
$$+(6x + 4y = 8)$$

$$8x = 16$$

$$x = 2$$

Sub in ①

29

$$2x^2 - 3y^2 = 5xy$$

$$-3x + y = 5$$

Make y the subj of ②

$$y = 5 + 3x$$

Sub into ①

$$2x^2 - 3(5+3x)^2 = 5x(5+3x)$$

$$2x^2 - 3[25 + 30x + 9x^2] = 25x + 15x^2$$

$$2x^2 - 75 - 90x - 27x^2 = 25x + 15x^2$$

$$-40x^2 - 115x - 75 = 0$$

$$\div 5] \quad 8x^2 + 23x + 15 = 0$$

$$8x^2 + 8x + 15x + 15 = 0$$

$$8x(x + 1) + 15(x + 1) = 0$$

$$(x + 1)(8x + 15) = 0$$

Either

$$x = -1$$

When $x = 1$
$$y = 5 + 3(-1)$$
$$y = 5 - 3$$
$$y = 2$$
$$(-1, 2)$$

Or

$$x = \frac{-15}{8}$$

When $x = -15/8$
$$y = 5 + 3\left(\frac{-15}{8}\right)$$
$$y = -\frac{5}{8}$$
$$\left(-\frac{15}{8}, -\frac{5}{8}\right)$$

30

$$\frac{27^{2x-3}}{9^{x+4}} = 3$$

$$\frac{3^{3(2x-3)}}{3^{2(x+4)}} = 3^1$$

$$3^{3(2x-3) - 2(x+4)} = 3^1$$

Equate Indices

$$6x - 9 - 2x - 8 = 1$$

$$4x - 17 = 1$$

$$4x = 18$$

$$x = \frac{-18}{4}$$

$$x = \frac{-9}{2}$$

31

$$x \propto y^2$$

$$x = ky^2$$

$$62.5 = k(5)^2$$

$$k = \frac{62.5}{25}$$

$$k = 2.5$$

$$\therefore x = 2.5y^2$$

a)
$$\frac{125}{72} = 2.5y^2$$

$$\frac{25}{36} = y^2$$

$$y = \pm \frac{5}{6}$$

b)
$$x_1 = ky_1^2$$

when y doubled $\quad y_2 = 2y_1$

$$x_2 = k(2y_1)^2$$

$$x_2 = k \times 4y_1^2$$

$$x_2 = ky_1^2 \times 4$$

$$x_2 = x_1 \times 4$$

$\cdots$ x is multiplied by 4

c)
$$x = ky^2$$

when y is increased

by 80%. $\quad y_2 = 1.3y_1, \cdots$

$$x_2 = k(1.3y_1)^2$$

$$x_2 = k \times 1.69y^2$$

$$x_2 = ky^2 \times 1.69$$

$$x_2 = x_1 \times 169\%.$$

$\therefore$ x is increased by 69%

32

$$x \propto \frac{1}{y^3}$$

$$x = \frac{k}{y^3}$$

$$0.75 = \frac{k}{2^3}$$

$$k = 6$$

$$\therefore x = \frac{6}{y^3}$$

a)
$$x = \frac{6}{(-3)^3}$$

$$x = \frac{6}{-27}$$

$$x = -\frac{2}{9}$$

b)
$$x_1 = \frac{k}{y^3}$$

when y is doubled

$$x_2 = \frac{k}{(2y)^3}$$

$$x_2 = \frac{k}{8y^3}$$

$$x_2 = \frac{k}{y^3} \times \frac{1}{8}$$

$$x_2 = x_1 \times \frac{1}{8}$$

c) $x = \dfrac{k}{y^3}$

when y is decreased
by 10%. $y_2 = 0.9y$

$\therefore x_2 = \dfrac{k}{(0.9y)^3}$

$\therefore x_2 = \dfrac{k}{0.729y^3}$

$\therefore x_2 = \dfrac{k}{y^3} \times \dfrac{1}{0.729}$

$$\cancel{x_2} = \cancel{x_1} \times \cancel{1000 \over 729}$$

$x_2 \simeq x_1 \times 1.371$

$\therefore x$ is increased
by 37.1%.